

## Nucleation of cracks in a brittle sheet

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(Received 14 June 2009; published 11 December 2009)

We use molecular dynamics to study the nucleation of cracks in a two-dimensional material without pre-existing cracks. We study models with zero and nonzero shear moduli. In both situations, the time required for crack formation obeys an Arrhenius law, from which the energy barrier and prefactor are extracted for different system sizes. For large systems, the characteristic time of rupture is found to decrease with system size, in agreement with classical Weibull theory. In the case of zero shear modulus, the energy opposing rupture is identified with the breakage of a single atomic bond. In the case of nonzero shear modulus, thermally activated fracture can only be studied within a reasonable time at very high strains. In this case, the energy barrier is much higher compared to the zero shear modulus case. This barrier is understood within adiabatic simulations.

DOI: [10.1103/PhysRevE.80.066109](https://doi.org/10.1103/PhysRevE.80.066109)

PACS number(s): 62.20.mm, 62.20.mt, 64.60.av, 64.60.qe

### I. INTRODUCTION

While our current understanding of fracture begins with the ideas of Griffith in 1921 [1], the study of its atomic mechanism has attracted considerable attention in recent years. For example, corrections to Griffith's results for a crack in a brittle material have been proposed and verified with atomistic simulations [2–4]. Also, large scale simulations have been used to study dynamical fracture [5–7]. This increase in interest in fracture is partly due to computer simulations, which promise an understanding of the phenomenon at the atomic level. However, simulations face a fundamental problem [8,9]: many atomic deformations are thermally activated and therefore involve long time scales, which are difficult to simulate.

Most simulations overcome this problem by studying fracture with a pre-existing crack. In that case, the crack growth is a driven phenomenon and there is no energy barrier to be overcome. Only a few simulations have been used to study the formation of cracks at nonzero temperature without pre-existing cracks: void formation has been observed in three-dimensional simulations of strained binary Lennard-Jones systems [10], and simulations for the rate of crack nucleation have been performed in a two-dimensional (2D) spring network [11]. Experimentally, the rate of crack nucleation in heterogeneous materials has been found to obey an Arrhenius law with an energy barrier scaling according to Griffith's results [12,13].

In the present work, we address the nucleation of cracks in a brittle two-dimensional material, i.e., a sheet with a thickness of one atomic layer, through Langevin dynamics. The rate constant for the nucleation of cracks follows an Arrhenius law, from which the energy barrier is extracted. Two variants of a square lattice are studied: atomic interactions restricted to first neighbors and interactions extended to second neighbors. In the former case, the shear modulus of the solid is zero and the energy barrier is shown to be inde-

pendent of system size: the breakage of a single bond propagates to the rest of the solid without any cost. In the latter case, the shear modulus is nonzero and a finite-size crack has to nucleate before rupture can propagate throughout the system. These two situations will be referred to as chainlike and solidlike models, respectively.

This paper is organized as follows. The next section begins with a description of the models used in this paper followed by a description of how simulations are carried out. In Sec. IV we present the results for the chainlike and solidlike models. The latter is physically more relevant to describe brittle materials, and we discuss its energy barrier in the context of Griffith's theory in Sec. V.

### II. MODEL

A stretched one-dimensional chain has been previously used as a simple model for the breakage of polymers [14–20]. Here we extend this model to study fracture in 2D brittle solids by bonding the chains to each other so as to form a square lattice (see Fig. 1). We study samples containing  $M$  chains, which are made of  $N=100$  atoms each. Those chains are stretched in the horizontal direction, with their total length fixed at  $N(a+s)$ , where  $a$  is the equilibrium bond length and  $s$  is the applied strain. By constraining the atoms

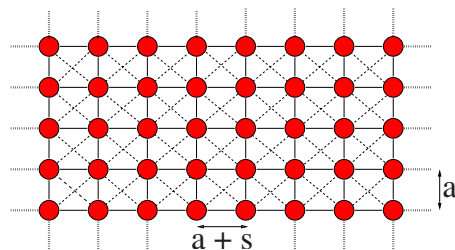


FIG. 1. (Color online) Schematic representation of a system containing  $M \times N = 5 \times 8$  atoms. Atoms are only allowed to move in the horizontal direction. First-neighbor interactions are represented by solid lines, while dotted lines represent second-neighbor interactions. Periodic boundary conditions are represented by dashed lines.

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to move only along the direction of the applied strain, we prevent the system from forming topological defects, and so the only mechanism for stress relaxation is fracture. Also, by choosing the constraint to allow motion only along the applied strain, we expect to be sampling the pathway for fracture while speeding up the simulation. We show that this constraint does not change the energy barrier in Sec. IV, where we perform one set of simulations without this constraint.

A square lattice can be made elastically isotropic by choosing the spring constant between first neighbors to be twice as large as the spring constant between second neighbors [21]. To fulfill this condition, we chose the following form for the Lennard-Jones potential between first neighbors:

$$V_f(r) = \epsilon [(a/r)^{12} - 2(a/r)^6], \quad (1)$$

and

$$V_s(r) = 4\epsilon_s [(a_s/r)^{12} - (a_s/r)^6], \quad (2)$$

for second neighbors. We use  $\epsilon=1$  and  $\epsilon_s=36\epsilon/228$  for the binding energies and  $r_o=a=1$  and  $r_{so}=2^{1/6}a_s=\sqrt{2}a$  for the equilibrium lengths. For this choice of parameters, the spring constant of the interaction between first neighbors  $k_f=72\epsilon/a^2$  is twice as large as the spring constant between second neighbors  $k_s=456\epsilon_s/a_s^2$ .

The dynamics of this system is obtained by solving a set of Langevin equations for the horizontal position  $x_{i,j}$  of each atom ( $i$  and  $j$  indicate the column and row of the atom in the lattice),

$$m \frac{d^2 x_{i,j}}{dt^2} = \sum_{k,l} F(x_{i,j} - x_{k,l}) - \eta \frac{dx_{i,j}}{dt} + f_{i,j}(t), \quad (3)$$

where  $F(x)$  is the force computed from the potential,  $m$  is the atomic mass, and  $\eta$  is the friction coefficient. The random force  $f_{i,j}(t)$  is related to  $\eta$  by the fluctuation-dissipation theorem.

Periodic boundary conditions in the horizontal direction imply  $x_{0,j}=x_{N,j}$  and  $x_{N+1,j}=x_{1,j}$  for all  $j$ . Periodicity is also imposed in the vertical direction to ensure that all chains are equivalent:  $x_{i,0}=x_{i,M}$  and  $x_{i,M+1}=x_{i,1}$ . For simplicity, we use reduced units. Energy is given in units of  $\epsilon$ , distance is given in terms of  $a$ , and time is given in units of the smallest phonon oscillation period  $P=2\pi/(12\sqrt{2}\epsilon/ma^2)$  of an intact chain [14]. Mass is written in terms of  $m$  and the friction coefficient is tuned to  $\eta=0.25(2\pi/P)$ . The temperature  $T$  of the system is given in units of  $\epsilon/k_b$ , where the Boltzmann's  $k_b$  is set to one.

Initially, all the horizontal bonds have the same length  $a+s$  and all vertical bonds are at their equilibrium length  $a$ . The velocity of each atom is chosen randomly according to the Boltzmann distribution. The dynamics of the system is obtained by solving numerically Eq. (3) using the velocity-Verlet algorithm [22] until the solid ruptures.

### III. SIMULATION

In Fig. 2 (upper panel), we show the time dependence of the potential energy for the solidlike model. At early times,

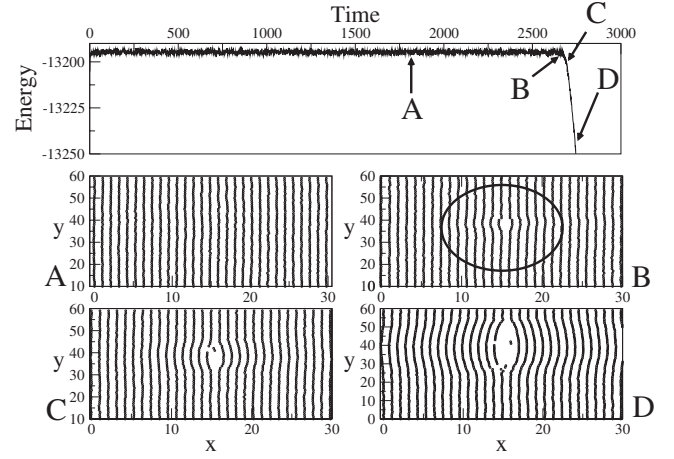


FIG. 2. (Upper panel) Dependence of the potential energy for the solidlike model defined by  $M \times N=60 \times 100$ ,  $s=0.065$ , and  $T=0.016$ . Arrows indicate the instances at which the atomic configurations in panels A–D are shown.

the potential energy fluctuates, as the system explores a metastable energy well. Rupture occurs at about 2750 units of time when the energy of the system drops abruptly. This shows that the system is been driven toward an equilibrium state with a lower energy. In panels A, B, C, and D of this figure, we show atomic configurations at different times during the fracture process. In those panels, the growing crack is seen to propagate in the direction perpendicular to the direction of applied strain. This indicates that we can use the sum of bond lengths along the surface at which fracture is taking place as an order parameter  $\phi$  for fracture. For convenience, the sum is taken over all the largest bonds percolating vertically along the sample and only the horizontal bonds are considered in the sum. Thus, initially  $\phi=M(a+s)$  and  $\phi$  increases until two surfaces are formed [31].

Notice that the potential energy in the upper panel of Fig. 2 shows no apparent precursor behavior for rupture. Also, the energy barrier that the system has to overcome for rupture to proceed is smaller than the fluctuations in the total potential energy. Thus, it cannot easily be extracted from an analysis of the potential energy. To obtain this barrier, we study the kinetics of the system as it proceeds toward rupture. In particular, we measure the characteristic time of rupture and analyze this quantity from the point of view of the theory of thermally activated systems.

To compute the characteristic time of rupture  $\tau$ , we use an ensemble containing  $S_0=1000$  samples for the chainlike model and  $S_0=500$  samples for the solidlike model. Those samples differ from each other by the initial velocities and random forces  $f_{i,j}(t)$ . We will need to choose a value of the order parameter  $\phi$ , which we will associate with irreversible rupture. To do this, the characteristic time for the incipient crack to reach a particular size is computed by tracking the number of samples  $S(\phi, t)$ , whose order parameter has not yet reached the value  $\phi$  at time  $t$ . For a fixed value of  $\phi$ , this number decreases exponentially with time

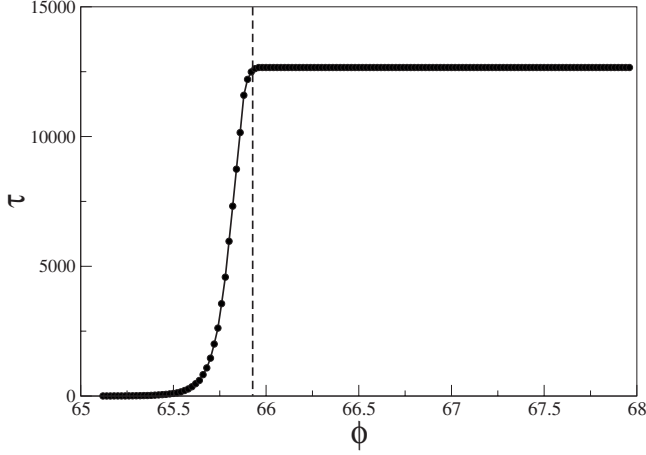


FIG. 3. Dependence of the characteristic time on the cut-off value of the order parameter for the solidlike model defined by  $M \times N = 60 \times 100$ ,  $s = 0.065$ , and  $T = 0.016$ . The dashed line separates reversible (left side) from irreversible (right side) rupture.

$$S(\phi, t) = S_0 \exp[-t/\tau(\phi)]. \quad (4)$$

The characteristic time  $\tau(\phi)$  depends on  $\phi$  and is obtained from a fit of  $S(\phi, t)$  to the numerical data.

The time at which irreversible rupture occurs depends on the arbitrary choice of the cutoff value  $\phi_c$ , which we associate with rupture. To choose this cutoff, we consider the dependence of  $\tau$  on  $\phi$  shown in Fig. 3. Two distinct regimes are apparent. The first regime occurs when  $\phi$  is smaller than approximately 65.8 (in units of  $a$ ). In this regime,  $\phi$  increases slowly with time. The underlying physics of this regime is the competition between thermal fluctuations, which are responsible for increasing crack length, and the restoring force on the atomic bonds. The second regime occurs when the order parameter  $\phi$  is greater than 65.8. Here,  $\tau(\phi)$  has reached a plateau, and  $\phi$  increases very rapidly with time. Stress relief in the bulk material is the driving force in this regime which therefore produces a fast increase in  $\phi$ : irreversible rupture has occurred. So, from Fig. 3, we can determine the value of  $\phi$  for which rupture becomes irreversible. This value is  $\phi_c = 65.8$ .

#### IV. RESULTS

In our simulations, the nucleation of cracks is thermally activated [8,9,11] such that their occurrence is typical of an Arrhenius process. The characteristic time of rupture  $\tau$ , the inverse of the nucleation rate, is

$$\tau = \tau_0 \exp(E_b/k_b T), \quad (5)$$

where  $k_b T$  is thermal energy,  $E_b$  is the energy barrier the system has to overcome, and  $\tau_0$  is the inverse of the attempt frequency. The attempt frequency depends on the vibration frequency of the system in the initial metastable well given by the local curvature of the energy surface [24,25]. It also depends on the friction coefficient in the Langevin equation [23]. In this section, we study the dependence of  $\tau$  on the temperature for the chainlike and solidlike models to extract

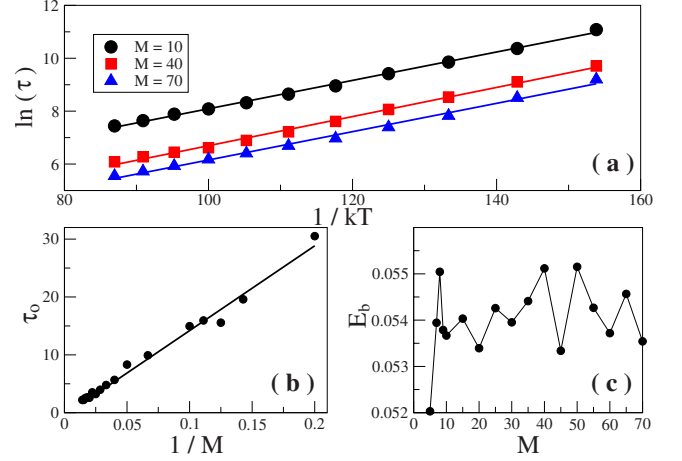


FIG. 4. (Color online) Chainlike model. (a) Dependence of  $\ln(\tau)$  on the inverse of temperature for different system sizes  $M$ . (b) Dependence of  $\tau_0$  on system size. (c) Dependence of the energy barrier  $E_b$  on system size.

both  $E_b$  and  $\tau_0$ , which are intrinsic quantities of the system being studied [26].

#### A. Chainlike model

In Fig. 4, we show the temperature dependence of  $\tau$  for different system sizes  $M$ . The strain is set to  $s = 0.05$ . For each system size,  $\tau$  increases exponentially with  $1/k_b T$ , in agreement with Eq. (5). The energy barrier  $E_b$  and the prefactor  $\tau_0$  are extracted from fits to those results. Changing system size strongly affects the prefactor but has no effect, within error bars, on the energy barrier, as can be seen in Figs. 4(b) and 4(c), respectively.

The dependence of the prefactor on system size can be understood qualitatively within the scope of a nucleation theory for fracture, the classical Weibull theory. It assumes that the probability  $S(t)/S_0$  of a sample of size  $M$  to survive without fracture at time  $t$  scales with the probability  $P(t)$  that a single chain will not fracture at time  $t$  to the power  $M$ . Note that  $S(t) \equiv S(\phi_c, t)$  was defined in Eq. (4). Equating the probability of nonfracture to  $S/S_0$  yields  $(e^{-te^{-BE_b}})^{1/\tau_0} = P^M$ . Figure 4(b) shows the good agreement of the simulation with this inverse relation  $\tau_0 = 1/M$ .

The energy barrier can be understood quantitatively by assuming that parallel chains are independent from each other. Under this assumption, the energetic cost  $E$  of elongating one atomic bond in a single chain is given by [14,15,17]

$$E(\phi) = V(a + s + \delta) + (N - 1)V\left(a + s - \frac{\delta}{(N - 1)}\right), \quad (6)$$

where  $\delta$  is the deviation of the broken bond length from its strained elongation and  $V(x)$  is the potential energy of an atomic bond. Equation (6) corresponds to the sum of potential energies of all the bonds in the chain containing the incipient fracture. This equation takes account of the fact that while one of the bonds increases toward rupture by an amount  $\delta$ , the other bonds of the same chain relax by an

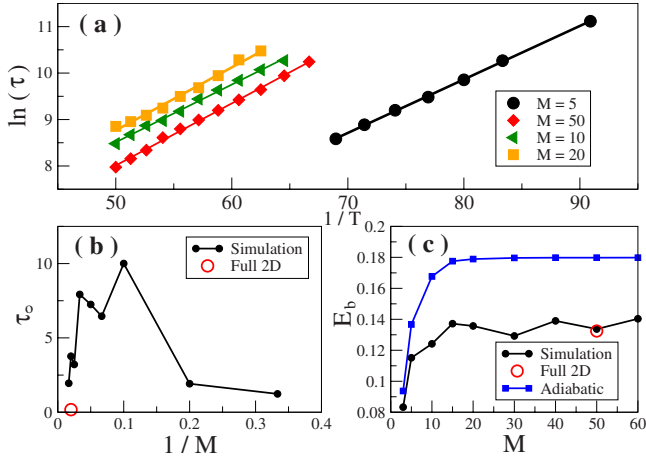


FIG. 5. (Color online) Solidlike model. (a) Dependence of  $\ln(\tau)$  on the inverse of temperature for different system sizes  $M$ . (b) Dependence of  $\tau_0$  on system size. (c) Dependence of the energy barrier  $E_b$  on system size. The results for the “full 2D” system correspond to a dynamically unconstrained model (see text for more details).

amount  $\delta/(N-1)$  toward their equilibrium value. For the parameters used in Fig. 4, i.e.,  $N=100$  and  $s=0.05$ , Eq. (6) predicts an energy barrier of 0.0564. This is in good agreement with our simulations, where the barrier is approximately 0.054 for all system sizes.

Independence of parallel chains is the key assumption to explain fracture in the chainlike model. This assumption can be understood as follows. Since the shear modulus of this model is zero, no energetic cost is associated with sheared configurations in the linear regime. Therefore, an individual chain can proceed toward fracture independently of neighboring chains until nonlinear effects become relevant. The energy barrier opposing this process is given by the competition between the energy costs of increasing the length of one of the bonds of the chain—independently of other chains—and the energy relaxation resulting from the bulk elastic forces.

Equation (6) results from the competition between the energetic cost of extending one bond length of the chain and the energetic gain of relaxing the remaining bonds. This contrasts with Griffith’s calculation, where the barrier is related to the cost of creating more surface and the energetic gain due to reducing the strain in the bulk of the material. Thus, despite its use in the literature, the square lattice with only first-neighbor interactions is a poor model for fracture in a solid and Griffith’s theory does not apply to this system.

### B. Solidlike model

We now study the solidlike model. In Fig. 5, we show the temperature dependence of  $\tau$  for different system sizes  $M$  and applied strain  $s=0.065$ . As in the previous model, we study this system by fitting the time of fracture to Eq. (5), obtaining the energy barrier  $E_b$  and the prefactor  $\tau_0$  for each system size. Those results are shown in Figs. 5(b) and 5(c).

The prefactor [Fig. 5(b)] presents two regimes: for systems containing less than 15 chains, i.e.,  $M < 15$ ,  $\tau_0$  increases with system size; however for  $M > 15$ , the prefactor de-

creases as system size increases. Those behaviors are related to finite-size effects. When  $M < 15$ , the relaxation region around the crack is of the same size as the system. On the other hand, increasing the size of the solid above 15 layers implies that more nucleation sites are available for rupture, and  $\tau_0$  decreases with  $M$ , as in the previous model [27,28].

In Fig. 5(c), we show the energetic cost for nucleating a crack in the solidlike model as a function of system size  $M$ . For solids smaller than  $M=15$ , the energy barrier increases considerably with system size [more than 150% in Fig. 5(c)], while for solids larger than  $M=15$ , the increase is only marginal and shows a saturation trend at  $E_b \sim 0.14$ —indicating that finite-size effects become negligible. This value is comparable to the adiabatic barrier, which is computed by extending the length of one bond in small steps and then fixing its length while the other bonds are relaxed at zero temperature. In this process, the energy increases until the critical crack is formed. The maximum energy seen in this process corresponds to the energy required to nucleate the crack at zero temperature. The adiabatic energy for the different system sizes are represented by squares in Fig. 5. Notice that the barrier obtained in our simulations is smaller than the adiabatic energy barrier by about 23%. A smaller simulated barrier compared to the adiabatic case has also been observed for one-dimensional systems [16]. A possible explanation for this discrepancy might be that a zero-temperature calculation does not account for entropy, which plays a role in the free energy, opposing rupture in system with multiple degrees of freedom [29,30].

One important simplification imposed in our model with respect to two-dimensional solids is the constraint of atomic dynamics to one dimension. However, by imposing this constraint along the direction of applied stress, we expect to be sampling the pathway for rupture of a 2D solid. To verify this statement, we performed a set of simulations on a  $M \times N = 50 \times 100$  system, where the constraint on the motion of atoms was removed. The results of those simulations are shown in Figs. 5(b) and 5(c) and are referred to as full 2D. Notice that the full 2D system has a much lower prefactor than our constrained system. A discrepancy in the prefactor is expected since it is related to the vibration of the system and therefore its dynamics, which differ in the two models. To explain why the prefactor of the full 2D system is smaller than the prefactor of the constrained system, one would have to apply a multidimensional Kramers’s calculation to our problem. This calculation was performed successfully to study rupture in a one-dimensional chain [14], but its application to the present model is beyond the scope of this paper. Despite the discrepancy in the prefactor, the energy barrier in our constrained model and the full 2D are equal within error bars. We are therefore confident that our constrained model can be used to study the energetic behavior of 2D solids. In the next section, we discuss the simulated energy barrier in the context of Griffith theory for rupture and adiabatic simulations.

### V. SOLIDLIKE MODEL AND GRIFFITH

The introduction of a crack of size  $L$  in a solid characterized by a Young’s modulus  $E$  and subjected to a stress  $\sigma$  will

result in a stress-energy relief of  $\pi\sigma^2L^2/2E$ . But this crack will also involve a cost of  $2\gamma L$ , where  $\gamma$  is the surface energy, such that the dependence of the energy on the crack size  $L$  is [13]

$$E_G(L) = -\frac{\pi L^2 \sigma^2}{2E} + 2\gamma L. \quad (7)$$

This potential energy reaches a maximum when  $\partial E_G(L)/\partial L=0$ . This occurs at the critical value  $L_G=2E\gamma/(\pi\sigma^2)$ . Beyond this crack length, the crack propagates spontaneously to reduce the bulk strain in the material until the solid is broken into two pieces. The barrier for crack nucleation occurs at this critical length  $E_G(L_G)=\frac{2\gamma^2E}{\pi\sigma^2}=\frac{2\gamma^2}{\pi s^2E}$ . The Young modulus of the solidlike model is  $E=77.91$  (in units of  $\epsilon_f/a^3$ ) and the energy necessary to create two surfaces  $2\gamma$  is equal to the energy of two weak and one strong bond per interatomic distance  $\gamma=0.6587$ . For a strain  $s=0.065$ , the Griffith energy barrier is  $E_G=0.837$ , that is, approximately six times the value obtained from our statistical simulation.

It is of no surprise that Griffith's calculation is not valid for large strains. First, in this highly stretched regime, linear elasticity theory is not valid. Second, for strains larger than 0.04, only one chain needs to be completely broken in order to initiate the rupture process. In other words, while in Griffith's regime the mechanism behind rupture is the competition between the formation of new surface and stress relaxation, the physics of rupture in the highly stressed regime is the competition between stress relaxation and bond stretching at the formation of the incipient crack.

To understand the range of validity of Griffith calculation, we performed adiabatic relaxation ( $T=0$ ) simulations in which bonds were mathematically cut along a line perpendicular to the applied strain. After cutting those bonds, atoms were relaxed until the force on each of them was smaller than  $1 \times 10^{-5}$ . To avoid finite-size effects, we increased the size of the system from  $N=100$  at  $S=0.04$  to  $N=600$  at  $S=0.01$ . The relative energy barrier computed from this process with respect to the Griffith barrier is shown in Fig. 6. This figure shows that for strains below 0.04, the adiabatic barrier differs by at most 30% from the Griffith barrier. On the other hand, for strains beyond 0.04 those barriers are several times different and this difference increases with strain. In the inset of Fig. 6, we also show in a log-log scale the dependence of the energy barrier on strain. For strains below 0.04, those quantities scale with an exponent of  $-2.26$ . This is very close to the exponent predicted by Griffith's theory:  $-2$ . The behavior at higher strains deviates from this scaling. This clearly shows that for our model, Griffith's

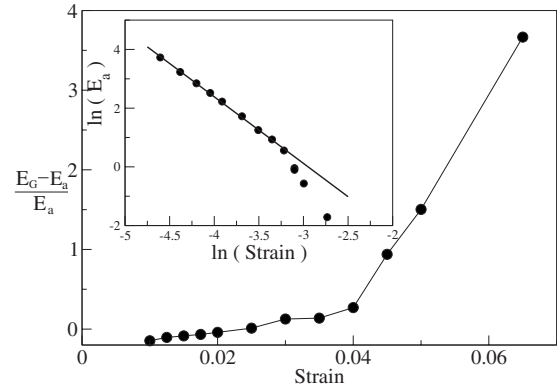


FIG. 6. Relative difference in energy barrier as computed from Griffith's calculation ( $E_G$ ) and the adiabatic simulation ( $E_a$ ):  $(E_G - E_a)/E_G$ . Inset: dependence of the adiabatic barrier on strain in a log-log scale. The linear fit to systems with a strain smaller than 0.04 is shown.

theory is valid for strains smaller than 0.04. We note that the calculation of the energy barrier with strain-dependent Young modulus and surface tension—as described in Ref. [3]—gives results in greater disagreement than the ones of Griffith.

## VI. CONCLUSION

Due to its simple dynamics and large system size, our atomistic simulation of the nucleation of cracks in thin brittle sheets is an ideal system for the study of noise-activated processes and nucleation theory. In particular, we found that the energy barrier for crack nucleation in a square lattice with only first-neighbor interactions is comparable to the barrier for the failure of a one-dimensional chain due to the zero shear modulus of this system. For the more interesting case, where second-neighbor interactions are incorporated into the model, we found agreement between the simulated energy barrier at high strains and the one computed from an adiabatic relaxation. This barrier is much higher compared to the case of isolated chains. We believe that extensions of the present study such as to investigate the growth of pre-existing cracks would be a valuable contribution to the understanding of fracture.

## ACKNOWLEDGMENTS

This work was supported by the Natural Sciences and Engineering Research Council of Canada and *le Fonds Québécois de la recherche sur la nature et les technologies*.

- [1] A. A. Griffith, Philos. Trans. R. Soc. London, Ser. A **221**, 163 (1921).  
 [2] M. Ippolito, A. Mattoni, L. Colombo, and Nicola Pugno, Phys. Rev. B **73**, 104111 (2006).

- [3] A. Mattoni, L. Colombo, and F. Cleri, Phys. Rev. Lett. **95**, 115501 (2005).  
 [4] M. Marder, Int. J. Fract. **130**, 517 (2004).  
 [5] M. J. Buehler, H. Tang, A. C. T. van Duin, and W. A. Goddard

- III, Phys. Rev. Lett. **99**, 165502 (2007).
- [6] D. Holland and M. Marder, Phys. Rev. Lett. **80**, 746 (1998).
- [7] M. J. Buehler, F. F. Abraham, and H. Gao, Nature (London) **426**, 141 (2003).
- [8] M. J. Alava, P. K. V. V. Nukala, and S. Zapperi, Adv. Phys. **55**, 349 (2006).
- [9] L. Golubovic and S. Feng, Phys. Rev. A **43**, 5223 (1991).
- [10] C. D. Lorenz and M. J. Stevens, Phys. Rev. E **68**, 021802 (2003).
- [11] S. Santucci, L. Vanel, A. Guarino, R. Scorretti, and S. Ciliberto, Europhys. Lett. **62**, 320 (2003).
- [12] A. Guarino, S. Ciliberto, and A. Garcimartaín, Europhys. Lett. **47**, 456 (1999).
- [13] A. Rabinovitch, M. Friedman, and D. Bahat, Europhys. Lett. **67**, 969 (2004).
- [14] A. Sain, C. L. Dias, and M. Grant, Phys. Rev. E **74**, 046111 (2006).
- [15] F. A. Oliveira and P. L. Taylor, J. Chem. Phys. **101**, 10118 (1994).
- [16] F. A. Oliveira, Phys. Rev. B **57**, 10576 (1998).
- [17] C. L. Dias, M. Dube, F. A. Oliveira, and M. Grant, Phys. Rev. E **72**, 011918 (2005).
- [18] A. Sain and M. Wortis, Phys. Rev. E **70**, 031102 (2004).
- [19] R. K. Puthur and K. L. Sebastian, Phys. Rev. B **66**, 024304 (2002).
- [20] C. F. Lee, Phys. Rev. E **80**, 031134 (2009).
- [21] L. Monette and M. P. Anderson, Modell. Simul. Mater. Sci. Eng. **2**, 53 (1994).
- [22] M. P. Allen and D. J. Tildesley, *Computer Simulation of Liquids* (Clarendon, Oxford, 1990).
- [23] H. A. Kramers, Physica (Utrecht) **7**, 284 (1940).
- [24] L. Angelani, G. Parisi, G. Ruocco, and G. Viliiani, Phys. Rev. E **61**, 1681 (2000).
- [25] G. Daldoss, O. Pilla, G. Viliiani, C. Brangian, and G. Ruocco, Phys. Rev. B **60**, 3200 (1999).
- [26] P. Hanggi, P. Talkner, and M. Borkovec, Rev. Mod. Phys. **62**, 251 (1990).
- [27] Z. P. Bazant and J. Planas, *Fracture and Size Effect in Concrete and Other Quasibrittle Materials* (CRC Press, Boca Raton, 1998).
- [28] W. Weibull, Proc. Royal Swedish Academy of Eng., Sci. **151**, 1 (1939).
- [29] J. Langer, Ann. Phys. (N.Y.) **54**, 258 (1969).
- [30] J. T. Pacia, L. Suna, T. Belytschkob, and G. C. Schatz, Chem. Phys. Lett. **403**, 16 (2005).
- [31] In practice, to compute the order parameter, we first measure the largest horizontal bond in the system. If this bond is in the  $i$ th chain, it is called  $\phi_i$ . Then, we compute the largest horizontal bond in the chain  $i+1$  that is either a first or second-nearest neighbor of the bond  $\phi_i$ . We call this bond  $\phi_{i+1}$ . This procedure is repeated for all the  $M$  chains in the system and the order parameter is cast mathematically as  $\phi = \sum_{i=1, M} \phi_i$ .